

Manipulators with Flexible Links: A Simple Model and Experiments

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1. Introduction

This paper briefly reviews a simple dynamic model proposed for flexible links, and presents experimental control results for different flexible systems. A simple dynamic model is useful for rapid prototyping of manipulators and their control systems, for possible application to manipulator design decisions, and for real-time computation as might be applied in model based or feedforward control. Such a model has been proposed, with the further advantage that clear physical arguments and explanations can be associated with its simplifying features and with its resulting analytical properties.

The model is mathematically equivalent to Rayleigh's method. Taking the example of planar bending, the approach originates in its choice of two amplitude variables, typically chosen as the link end rotations referenced to the chord (or the tangent) motion of the link. This particular choice is key in establishing the advantageous features of the model: its simplicity, its efficacy, its extensibility, its physical interpretability, its observability, and its controllability. A laboratory manipulator of modular design was constructed to permit rapid link changeout and overall reconfiguration, and was used to support the series of experiments reported here.

2. Background

Multiple link manipulators are characterized by non-linear relationships between displacements in the inertial frame and displacements (rotations) in the actuator (joint) space. Inertial forces exist which are thereby non-linear and cross-coupled with joint rotations even for manipulators with perfectly rigid links. On a manipulator with flexible links *any* inertial forces have the effect of further inducing deformations (and motions) which, in their simplest form, manifest themselves as vibrations which can easily exceed in magnitude the gross intended motions of the manipulator. The configuration-dependent conditions are essentially absent in the single-link systems which have attracted much of the research attention, but our interest is in an approach which is readily extended to multi-link, three-dimensional manipulators. Moreover, multi-link manipulators are characterized by high joint masses and inertia, and a dynamic model must readily handle these concentrated mass conditions as well.

We use the phrase *dynamic model* to refer to the construct by which the *equations of motion* are to be established. Taking as an example the most simple case of an elastic prismatic link with distributed mass in

planar bending, the true equations of motion are partial differential equations which in general are not solved explicitly. Rather, an approximate solution is used to express the elastic displacement with respect to the spatial variable, yielding a set of ordinary differential equations with respect to time, which constitute the *equations of motion*. A common approach is to form the *free vibration mode shapes* and use a truncated series to produce a tractable set of *equations of motion*. However, that approach is not ideally suited to the extended set of issues cited in the preceding paragraph. The proposed simple dynamic model has features which make it well suited to these extensions, as well as being easy in application and in understanding.

3. Description of the Simple Dynamic Model

The proposed model is described elsewhere [2] and its full presentation is not repeated here. The basic physical arguments can be formulated by referring, for the purposes of discussion, to the single planar link pictured in Figure 4. The rotation θ is a tangent to the link rotation (alternately θ may denote the rotation of the *chord* between the link end-points); two further variables are denoted, ϕ and ψ , constituting the link end rotations with respect to the chord. Two physical approximations are then made:

- The kinetic energy of the link distributed mass, m , is approximated as that of a mass m translating with the center-of-mass of the link *chord* itself.
- The displacement shape, with respect to the *chord*, is approximated as the displacement accompanying static end rotations ϕ and ψ . (The differential equation governing flexure is readily solved to yield the polynomial solution for the displacement.)

This model readily represents translations and rotations of the concentrated masses as linear combinations of θ , ϕ , and ψ . Moreover, the three rotation variables are readily observable through rotation and strain sensing, and are directly coupled to the actuator inputs.

In essence, the model has introduced two *amplitude variables* (ϕ and ψ) to approximate the elastic effects. The physical nature of most vibrations is such that this choice generally models the most significant vibration effects. Moreover, the formulation is expressly compatible with assembly of equations for multi-link systems, such that the most important configuration dependencies will be modelled automatically. Its assembly and its solution (computation) are simple, and as stated above the model variables are well-matched to the control problem.

In the general case, a rigid link has its position (with respect to its local origin) expressed by three rotations. For a flexible prismatic link, the model poses the need for eight rotations; three equivalent to the rigid body rotations, two for flexural end rotations in each of the two principal directions, and one (the *relative rotation* between the end points) in torsion. In our opinion the model will be reasonably effective at approximating the *equations of motion*¹.

¹The description applies most clearly to links which are prismatic, doubly symmetric, etc. For links which are irregular the physical reasoning used in posing the model can still apply, but the mathematics describing the displacement states must be updated; one of the experimental studies described in this paper includes such an extension, for a link of tapering cross-section.

4. Manipulator Configurations for Preliminary Experiments

A series of preliminary experiments have been performed at Carnegie-Mellon. The first was a single link in planar motion actuated by 1-DOF end rotation. The flexible link was of constant cross section with distributed mass and with concentrated mass at the tip, and the actuator was a small direct drive DC motor. The results of that experiment are described in an earlier paper [2] (with additional authors) and are not reproduced here; as expected, comparison of simulated and experimental histories confirmed a reasonable accuracy for the simple dynamic model.

The subsequent preliminary experiments in three-dimensional motion, described for the first time in this paper, were performed using a manipulator of modular design built at Carnegie-Mellon. The system features six actuators which connect through endplates and fittings to a variety of different links. This results in rapid changeout and inexpensive link fabrication; in addition to the experiments using flexible links the system has been used as a 6-DOF manipulator with rigid links operating under position control, and as a 4-DOF manipulator (using totally different configuration and link dimensions) with strain-sensing on the links operating under a supervisory level of force control. Each actuator consists of a DC motor, harmonic drive gearing, and a potentiometer for rotation sensing.

Figure 1 is a sketch of the manipulator as it was configured for the flexible link experiments. Three actuator units create a *roll, pitch, yaw* set of actuated DOF. Two flexible links were used. The first is pictured as a "fishing rod" with a tip mass; it was actuated through 2-DOF (*pitch* and *yaw*) and is further depicted in Figure 2. The second is pictured as a (flexible) "pipe" attached to a second (rigid) link; it was actuated through all 3-DOF and is further depicted in an analytical equivalent in Figure 3.

5. Results of 2-DOF Experiments

The motion of the "fishing rod" under 2-DOF actuation is a three-dimensional motion through a spherical angle, and a pilot experiment was first performed successfully by Heller [1]. The fishing rod is modelled here as a single link for which motion about the roll axis (torsional vibration) can be ignored. Figure 4 is the model of the link for motion about the pitch axis. Note that the link is of tapering cross section, and that in Figure 4 the end rotations ϕ and ψ are referenced to θ , the tangent to the link motion about its base. The inertial and friction properties of the actuators were determined by measurement and by system identification (not shown). The simple model was then applied to the link using the physical assumptions expressed earlier, and including without difficulty the variation in the cross section with length (also not shown, owing to requirements of brevity in this paper).

The basic experiment was a step motion (0.2 radians pitch rotation and 0.1 radians yaw rotation) performed under position control² only. Strain histories in Figure 5 evidence the resulting vibrations, and (with various other experimental observations) show the motions to be largely independent (uncoupled) of one another. The experiments were repeated adding feedback control on the strains at the base of the link. The experiment was performed about one particular point in joint space, and gains were chosen by trial and error. Figure 6 shows the resulting rotation and strain histories, evidencing an adequate reduction of vibration.

In Figures 5 and 6 the simulated histories were generated using the results of the simple dynamic model.

²Throughout this paper *position control* or *position feedback* refers to direct feedback control of *joint rotations*.

The model appears to be reasonably accurate in predicting the system frequencies. While the model was not used to set gains in this particular case, it did establish that the joint (actuator) rotations and link base strains would constitute the required state variables for control.

6. Results of 3-DOF Experiments

The configuration for the 3-DOF experiment can be considered a two-link system (proximal link is flexible and distal link is rigid) under general three-dimensional motion which will display coupled lateral-torsional vibrations. In the first experiment the joints were clamped to behave as a rigid boundary, and the system was set into motion by being given an initial tip displacement and being released at time zero. The resulting strain histories are shown in Figure 7a. They reveal the coupling of lateral and torsional vibrations, the significant vibration amplitudes, and the minimal material damping. The system was then restored to a configuration for position and strain feedback using gains chosen by trial and error for control about that point; Figure 7b shows the effective control of all vibrations under the same experimental excitation. Figure 8a shows the rotation and strain histories under a three-dimensional 3-DOF step motion, for the case of position feedback. Significant vibrations are observed; the damping present in this case (as compared to the results in Figure 7a) results from the dynamics (and friction) of the actuators. Figure 8b shows the rotation and strain histories when strain feedback feedback (on three channels of strain taken at the base) is added, evidencing effective vibration control. Analytical studies of the 3-DOF experiments have not yet been completed.

7. Summary

The approach and physical arguments for our simple dynamic model have been discussed briefly. The proposed model has various features which are well matched to demands which surface when studying manipulators with flexible links. At this time the model is proposed for the attention, consideration, and use of researchers. A series of experiments were performed demonstrating vibration suppression using direct feedback control on end rotations and link strains. For one experiment in which analytical results have been generated, the comparison of experimental and simulated histories shows reasonable performance of the simple dynamic model in capturing system frequencies and in confirming the needed state variables.

8. References

- [1] Heller, M.
Experiments in Feedback Control of a Flexible Link.
Master's thesis, Carnegie-Mellon University, 1988.
- [2] Shimoyama, I., Miura, H., Komatsu, T., and Oppenheim, I.
A Simple Dynamic Model for Control of Flexible Manipulators.
International Journal of Robotics Research under review(currently), 1987.

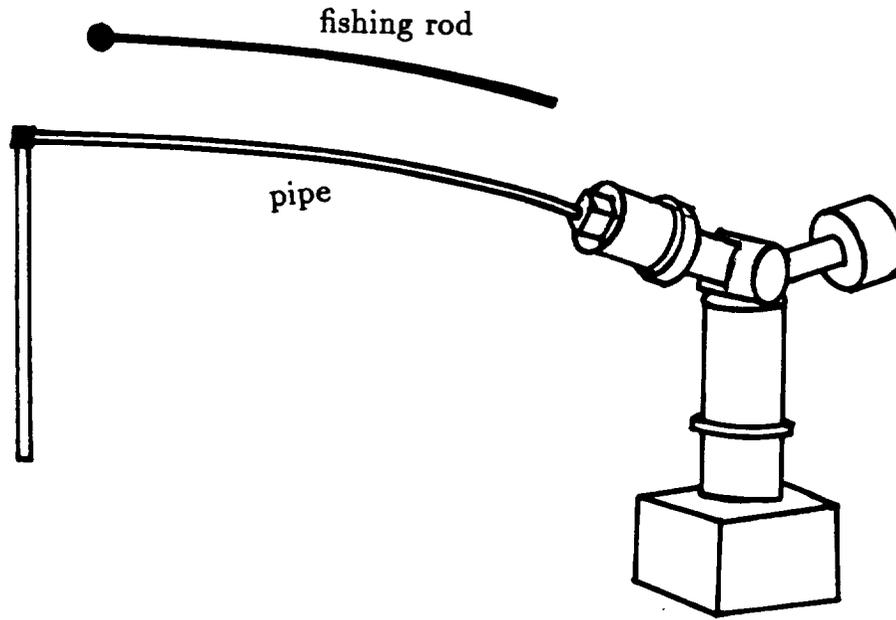


Figure 1. Manipulator as configured for 2-DOF (spherical motion, "fishing rod") and 3-DOF (coupled lateral-torsional motion, "pipe") experiment.

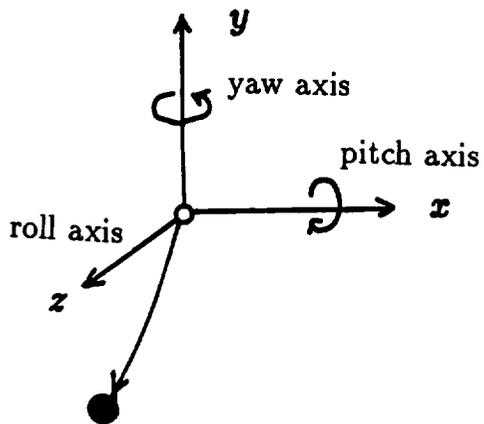


Figure 2. 2-DOF Experiment

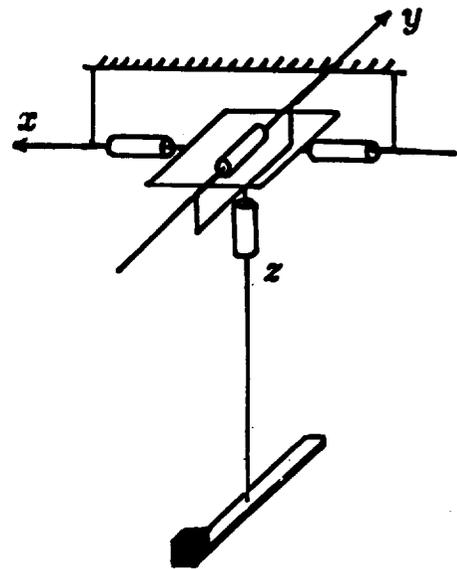
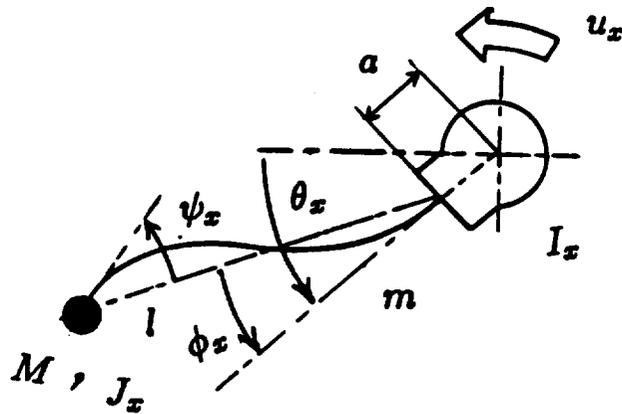


Figure 3. 3-DOF Experiment



- M, m mass
- I, J moment of inertia
- l, a length
- θ, ϕ, ψ angle
- u torque
- d_0 diameter at the base end of the fishing rod
- d_1 diameter at the tip end of the fishing rod

Figure 4. System Variables in the 2-DOF Experiment. (Pitch axis shown)

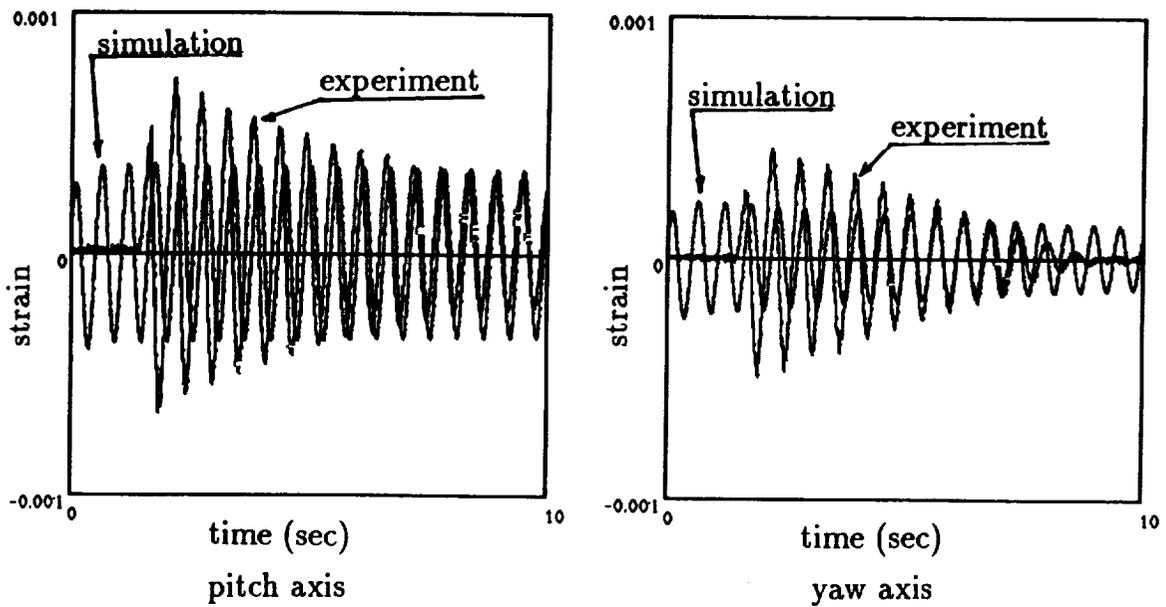


Figure 5. Strain Histories after a Step Motion, Position Feedback Only.

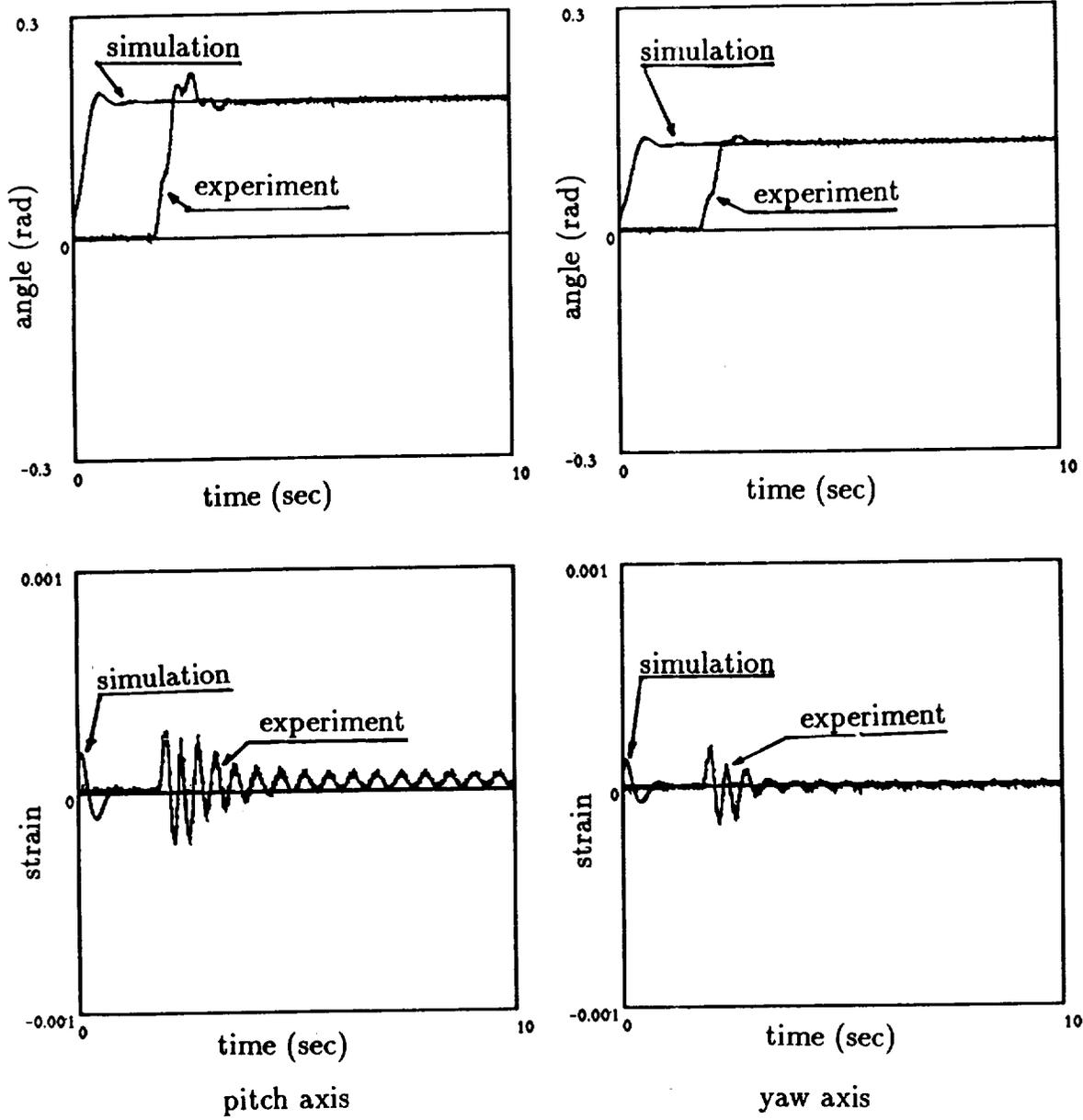


Figure 6. Rotation and Strain Histories after a Step Motion.
Position and Strain Feedback.

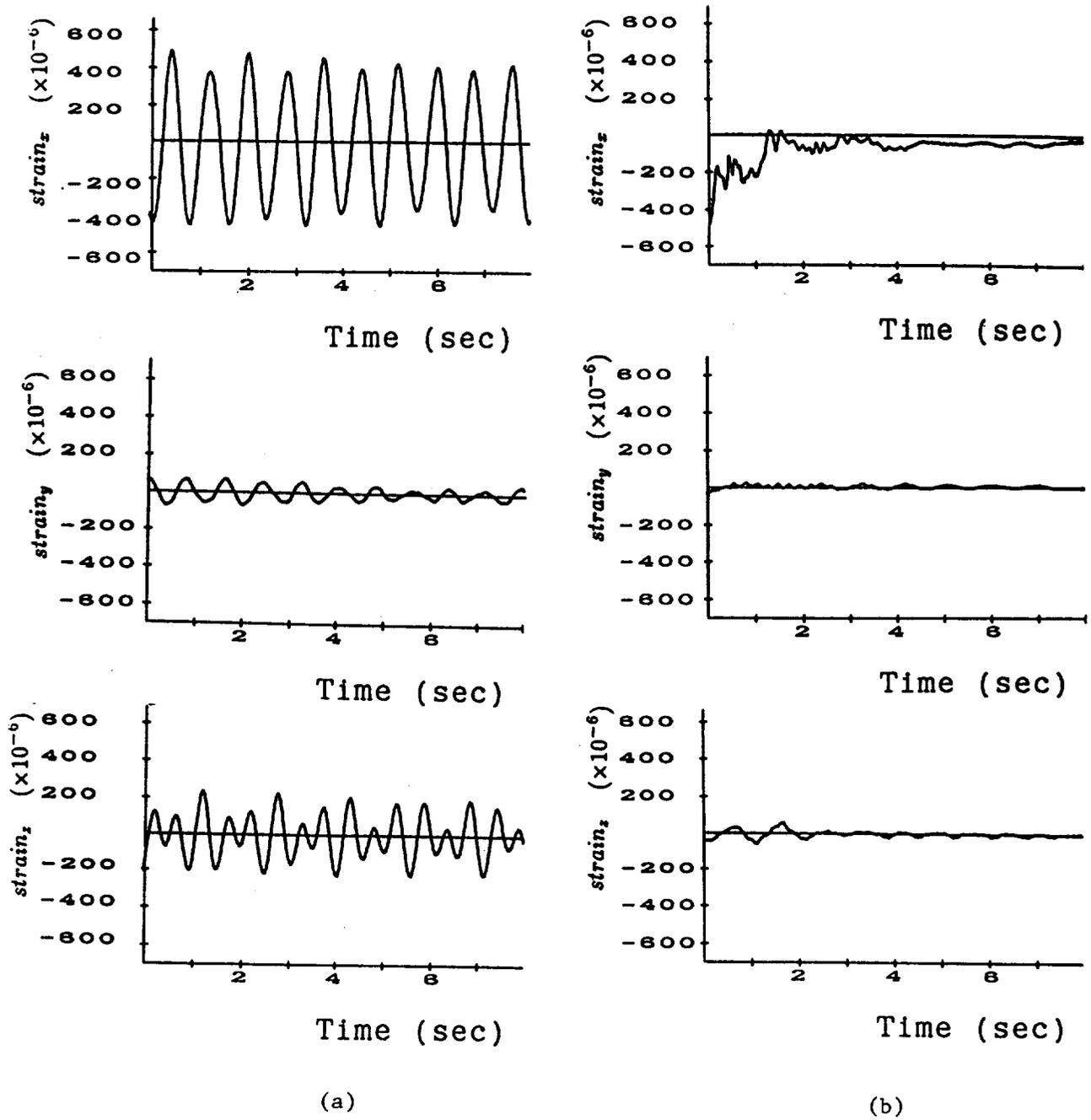


Figure 7. Strain Histories after Release from an Initial Tip Displacement
 (a) Three joints rigidly fixed; coupled lateral-torsional vibrations present
 (b) Three joints under position and strain feedback; vibrations eliminated

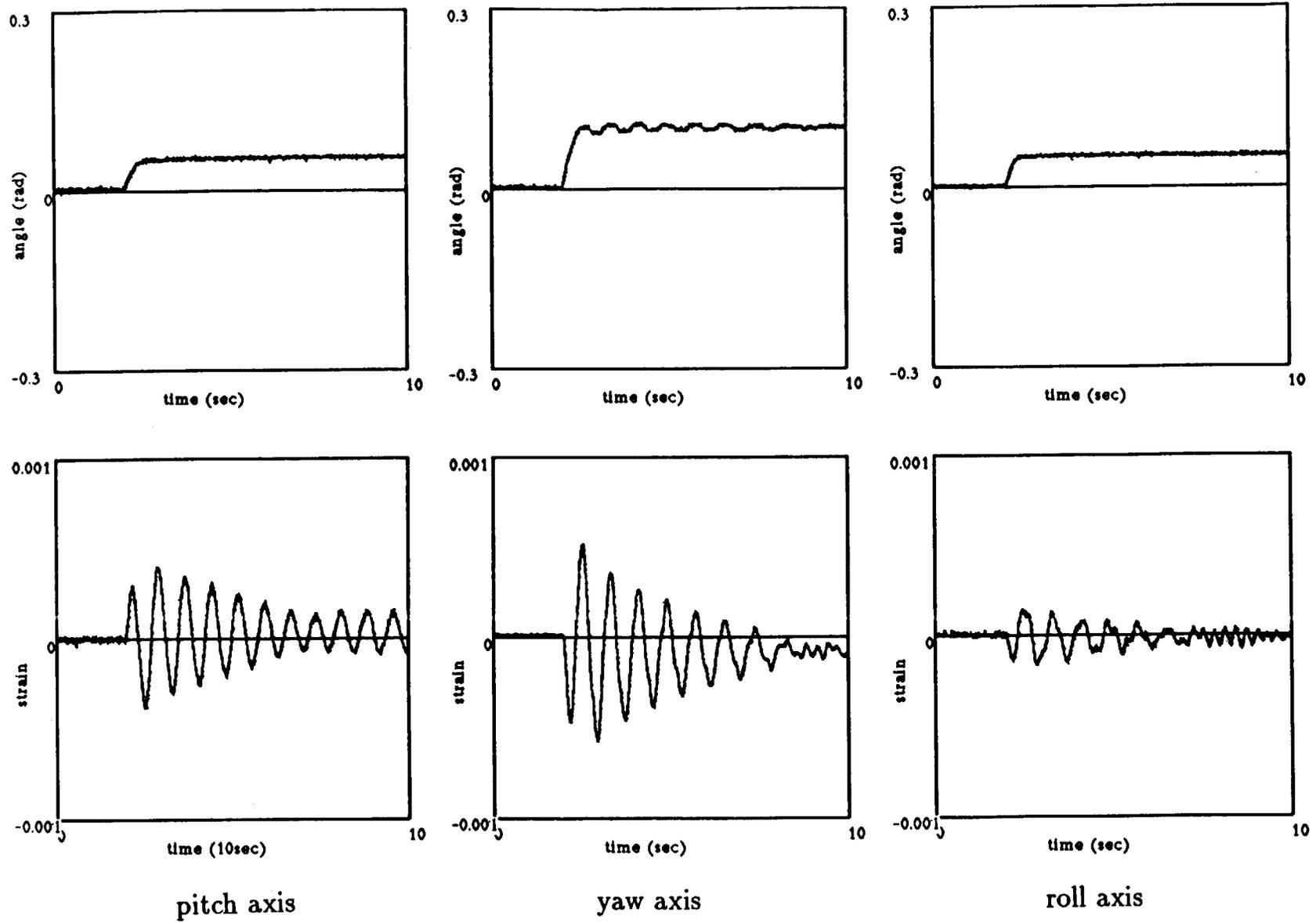


Figure 8a. Rotation and Strain Histories after a Step Motion, Position Feedback Only.

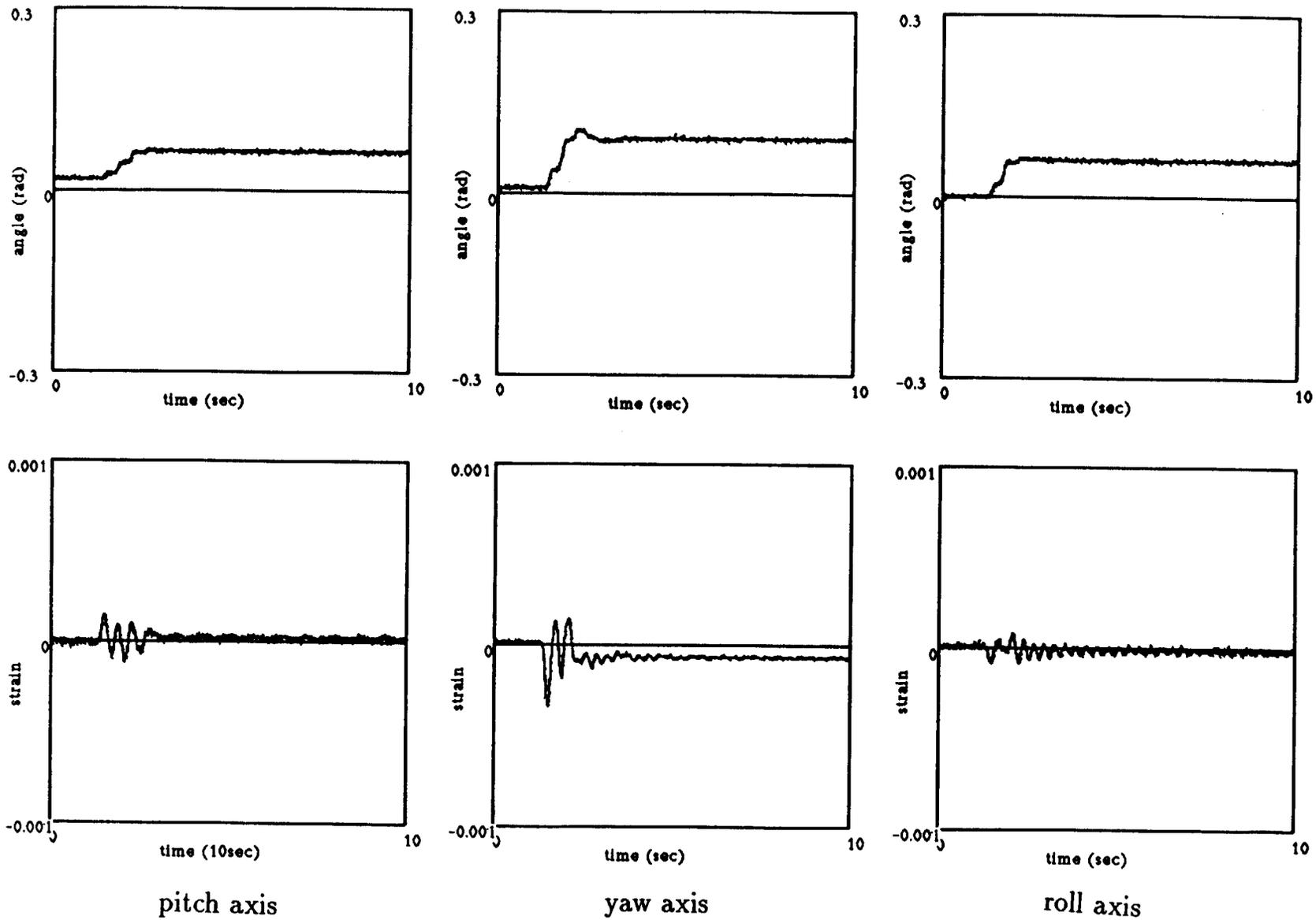


Figure 8b. Rotation and Strain Histories after a Step Motion, Position and Strain Feedback.